Exercise 71

Between 0°C and 30°C, the volume V (in cubic centimeters) of 1 kg of water at a temperature T is given approximately by the formula

$$V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3$$

Find the temperature at which water has its maximum density.

Solution

The domain of the function is $0 \le T \le 30$. Take the derivative.

$$\begin{aligned} V'(T) &= \frac{d}{dT}(999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3) \\ &= 999.87(0) - 0.06426(1) + 0.0085043(2T) - 0.0000679(3T^2) \\ &= -0.06426 + 0.0170086T - 0.0002037T^2 \end{aligned}$$

Set $V'(T) = 0$ and solve for T.

$$-0.06426 + 0.0170086T - 0.0002037T^2 = 0$$

$$T = \frac{-0.0170086 \pm \sqrt{(0.0170086)^2 - 4(-0.0002037)(-0.06426)}}{2(-0.0002037)}$$

$$T = 3.96651 \quad \text{or} \quad T = 79.5318$$

$T = 3.96651$ is within the interval $0 \le T \le 30$, so evaluate the function here.

$$\begin{aligned} V(3.96651) &= 999.87 - 0.06426(3.96651) + 0.0085043(3.96651)^2 - 0.0000679(3.96651)^3 \\ &\approx 999.745 \text{ cm}^3 \quad \text{(absolute minimum)} \end{aligned}$$

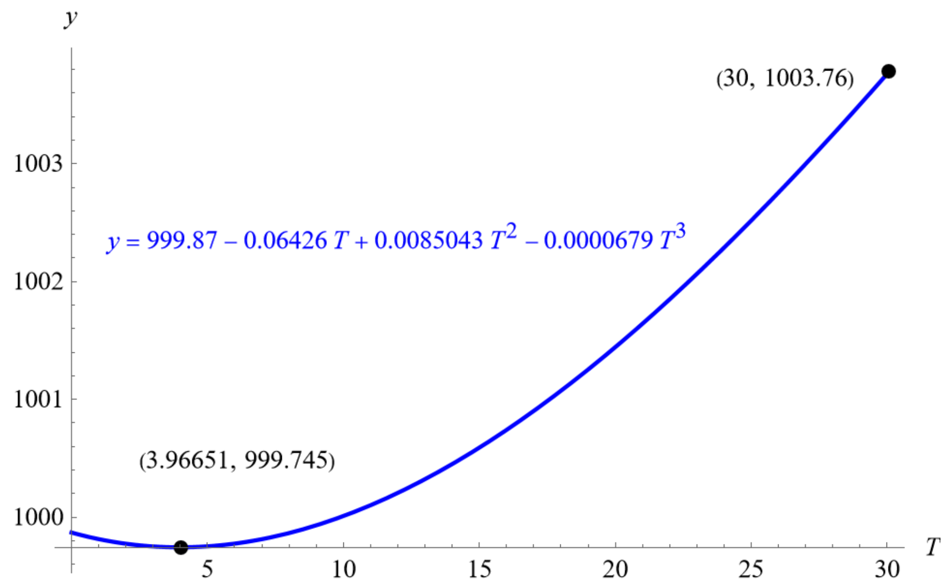
Evaluate the function at the endpoints.

$$V(0) = 999.87 - 0.06426(0) + 0.0085043(0)^2 - 0.0000679(0)^3 = 999.87 \text{ cm}^3$$

$$\begin{aligned} V(30) &= 999.87 - 0.06426(30) + 0.0085043(30)^2 - 0.0000679(30)^3 \\ &\approx 1003.76 \text{ cm}^3 \quad \text{(absolute maximum)} \end{aligned}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $0 \le T \le 30$.

The graph below illustrates these results.



Density is mass over volume:

$$\rho = \frac{m}{V}.$$

The density is at a maximum when the volume is at a minimum. Therefore, when the temperature is

$$T \approx 3.96651^{\circ}\text{C},$$

the density is at a maximum.